

Math (Science)	Group-I	Paper-I
Time: 2.10 Hours	(Subjective Type)	Max. Marks: 60

SECTION-I

2. Write short answers to any SIX (6) questions: 12

(i) Define Singular and Non-singular matrix.

Ans **Singular matrix:**

A square matrix A is called singular if the determinant of A is equal to zero.

Non-Singular matrix:

A square matrix A is called non-singular if the determinant of A is not equal to zero. $|A| \neq 0$.

(ii) If $\begin{bmatrix} a+3 & 4 \\ 6 & b-1 \end{bmatrix} = \begin{bmatrix} -3 & 4 \\ 6 & 2 \end{bmatrix}$, then find the value of a and b.

Ans

$$\begin{bmatrix} a+3 & 4 \\ 6 & b-1 \end{bmatrix} = \begin{bmatrix} -3 & 4 \\ 6 & 2 \end{bmatrix}$$

$$\begin{array}{l|l} a+3 = -3 & b-1 = 2 \\ a = -3-3 & b = 2+1 \\ a = -6 & b = 3 \end{array}$$

(iii) Describe in brief the concept of radical and radicand.

Ans If 'n' is a positive integer greater than one then $\sqrt[n]{a}$ is known as n^{th} root radical and a is called radicand.

(iv) Simplify: $(x^3)^2 \div x^{32}$.

Ans

$$\begin{aligned} (x^3)^2 \div x^{32} &= x^6 \div x^9 \\ &= \frac{x^6}{x^9} = x^{6-9} = x^{-3} \\ &= \frac{1}{x^3} \end{aligned}$$

(v) Find value of x : $\log_{625} 5 = \frac{1}{4} x$.

Ans $\log_{625} 5 = \frac{1}{4} x$

$$\begin{aligned}(625)^{1/4x} &= 5 \\ (5^4)^{1/4x} &= 5^1 \\ 5^x &= 5^1 \\ x &= 1\end{aligned}$$

(vi) Find value of x : $\log x = 0.0044$

Ans $\log x = 0.0044$
 $x = \text{Antilog}(0.0044)$
 $x = 1.01$

(vii) Which laws of surds are used to multiply and divide surds?

Ans The surds of same degree can be multiply and divide with each other. e.g.,

(i) ${}^n\sqrt{a} \times {}^n\sqrt{b} = {}^n\sqrt{ab}$

(ii) $\frac{{}^n\sqrt{a}}{{}^n\sqrt{b}} = {}^n\sqrt{\frac{a}{b}}$

(viii) Rationalize the denominator: $\frac{2}{\sqrt{5}-\sqrt{3}}$

Ans $\frac{2}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$
 $= \frac{2(\sqrt{5}+\sqrt{3})}{(\sqrt{5})^2 - (\sqrt{3})^2} = \frac{2(\sqrt{5}+\sqrt{3})}{5-3}$
 $= \frac{2\sqrt{5}+\sqrt{3}}{2} = \sqrt{5}+\sqrt{3}$

(ix) Factorize: $1 - 125x^3$

Ans $1 - 125x^3$
 $= (1)^3 - (5x)^3$
 $= (1 - 5x)((1)^2 + (1)(5x) + (5x)^2)$
 $= (1 - 5x)(1 + 5x + 25x^2)$

3. Write short answers to any Six (6) questions: 16

(i) Find L.C.M $39x^7y^3z$, $91x^5y^6z^7$

Ans $39x^7y^3z = 3 \times 13 \times x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot y \cdot y \cdot y \cdot z$
 $91x^5 \cdot y^6 \cdot z^7 = 7 \times 13 \cdot x \cdot x \cdot x \cdot x \cdot x \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y \cdot z \cdot z \cdot z$
H.C.F = $13 \times x^5 \cdot y^3z$

$$\text{N.C.F.} = 3 \times 7 \times x^2 y^3 \cdot z^6$$

$$\begin{aligned} \text{L.C.M} &= 3 \times 7 \times 13 \times x^5 \cdot x^2 \cdot y^3 \cdot y^3 \cdot z \cdot z^6 \\ &= 273 x^7 y^6 \cdot z^7 \end{aligned}$$

(ii) Define linear equation and write down its standard form.

Ans A linear equation in one unknown variable x (occurring to the first degree) is an equation of the form.

$$ax + b = 0 \quad \text{where } a, b \in \mathbb{R} \text{ and } a \neq 0.$$

(iii) Solve : $\sqrt{x-3} - 7 = 0$

Ans $\sqrt{x-3} = 7$

$$(\sqrt{x-3})^2 = (7)^2$$

$$x - 3 = 49$$

$$x = 49 + 3$$

$$x = 52$$

Check:

$$\sqrt{52-3} - 7 = 0$$

$$\sqrt{49} - 7 = 0$$

$$7 - 7 = 0$$

$$0 = 0$$

$$\therefore \text{S.S} = \{52\}$$

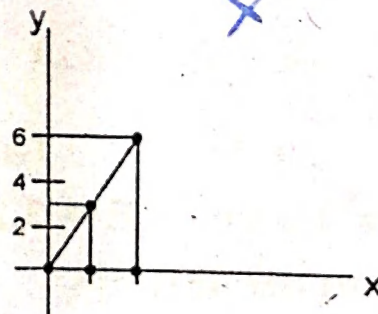
(iv) Define coordinate plane.

Ans Cartesian plane is also known as coordinate plane.

(v) Draw the graph : $y = 3x$.

Ans $y = 3x$

$x =$	0	1	2	3
$y =$	0	3	6	9



(vi) What are you meant by $\text{S.A.A} \cong \text{S.A.A}$?

Ans In any correspondence of two triangles if one side and any two angles of one triangle are congruent to the corresponding side and angle to the other triangle. The two triangles are congruent to each other.

$$(A.S.A \cong A.S.A)$$

(vii) Define scalene triangle.

Ans A triangle is called a scalene triangle if measures of all the three sides are different.

(viii) Find the mid-point: A (-4, 9), B (-4, 3).

Ans A (-4, 9), B (-4, -3)

$$P(x, y) = \left(\frac{-4 - 4}{2}, \frac{9 - 3}{2} \right)$$

$$P(x, y) = (-4, 3)$$

$$\text{Mid-point of AB} = (-4, 3)$$

(ix) Define parallelogram.

Ans A figure formed by four non-collinear points in the plane is called parallelogram.

1. Its opposite sides are of equal measure.
2. Its opposite sides are parallel.
3. Measure of none of the angles is 90° .

4. Write short answers to any SIX (6) questions: 12 (iv)

(i) Where will be the centre of a circle passing through three non-collinear points? (v)

Ans The point of concurrency of three right bisector is the centre of three non-collinear points. (v)

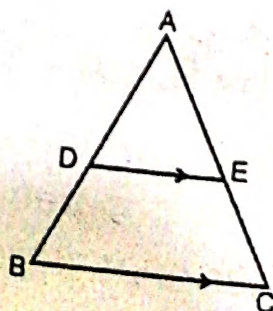
(ii) Why 2 cm, 3 cm and 5 cm cannot be the sides of a triangle? (v)

Ans Let

$$\begin{aligned} AB &= 2 \text{ cm} \\ BC &= 3 \text{ cm} \\ CA &= 5 \text{ cm} \\ AB + BC &> CA \\ 2 + 3 &> 5 \\ 5 &\neq 5 \end{aligned}$$

Not possible.

(iii) If $\overline{AD} = 2.4 \text{ cm}$, $\overline{AE} = 3.2 \text{ cm}$, $\overline{DE} = 2 \text{ cm}$, $\overline{BC} = 5 \text{ cm}$, then find \overline{AB} and \overline{DB} . (vi)



X

Ans

$$AE = \frac{1}{2} AC$$

$$\begin{aligned}\overline{AC} &= 2 \overline{AE} \\ &= 2 (3.2)\end{aligned}$$

$$\overline{AC} = 6.4 \text{ cm}$$

$$\begin{aligned}\overline{AB} &= \overline{AC} \\ &= 6.4 \text{ cm}\end{aligned}$$

$$\begin{aligned}\overline{DB} &= \frac{1}{2} \overline{AB} \\ &= \frac{1}{2} (6.4)\end{aligned}$$

$$\overline{DB} = 3.2 \text{ cm}$$

(iv) Define proportion.

Ans Equality of two ratios is defined as the proportion.

if $a : b = c : d$, then $a : b : c$ and d are said to be a proportion.

(v) Three sides of a triangle are measure 8, x and 17, respectively. For what value of x will it become base of a right angle triangle?

Ans To find x using

$$x^2 = (17)^2 - (8)^2$$

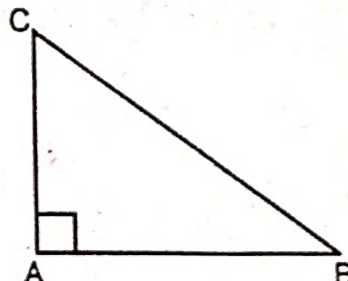
$$= 289 - 64$$

$$x^2 = 225$$

$$\sqrt{x^2} = \sqrt{225}$$

$$x = 15 \text{ cm}$$

(vi) In a right angled triangle having angle A as 90° , then:



(i) $\overline{AB}^2 = \dots\dots\dots$

(ii) $\dots\dots\dots = \overline{BC}^2 - \overline{AB}^2$

Ans (i) $\overline{AB}^2 = |\overline{BC}|^2 - |\overline{CA}|^2$

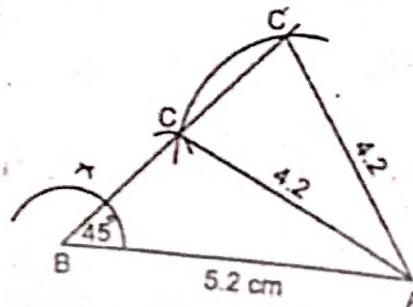
(ii) $|\overline{CA}|^2 = |\overline{BC}|^2 - |\overline{AB}|^2$

(vii) The area of a parallelogram is equal to that of rectangle on the same base and having same altitude.

Ans Parallelogram on equal bases and having the same (or equal) altitude are equal in area.

(viii) Construct $\triangle ABC$, where $m \overline{AC} = 4.2$ cm, $m \overline{AB} = 5.2$ cm, $m \angle B = 45^\circ$.

Ans



(ix) What do you mean by point of concurrency?

Ans The common point is called the point of concurrency of the lines.

SECTION-II

NOTE: Attempt any Three (3) questions. But question No. 9 is compulsory.

Q.5.(a) Solve the linear equation by the matrix Inverse method. (4)

$$3x - 2y = -6$$

$$5x - 2y = -10$$

Ans
$$\begin{bmatrix} 3 & -2 \\ 5 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -6 \\ -10 \end{bmatrix}$$

$$Ax = B$$

$$|A| = \begin{vmatrix} 3 & -2 \\ 5 & -2 \end{vmatrix}$$

$$|A| = -6 + 10$$

$$|A| = 4$$

$$A^{-1} = \frac{\text{Adj. } A}{|A|}$$

$$= \frac{\begin{bmatrix} -2 & 2 \\ -5 & 3 \end{bmatrix}}{4}$$

$$X = A^{-1}B$$

$$= \frac{1}{4} \begin{bmatrix} -2 & 2 \\ -5 & 3 \end{bmatrix} \begin{bmatrix} -6 \\ -10 \end{bmatrix}$$

$$= \frac{1}{4} \begin{bmatrix} 12 - 20 \\ 30 - 30 \end{bmatrix}$$

$$= \frac{1}{4} \begin{bmatrix} -8 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-8}{4} \\ \frac{0}{4} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \end{bmatrix}$$

$$x = -2$$

$$y = 0$$

(b) Solve the equation for real x and y: (4)

$$(2 - 3i)(x + yi) = 4 + i$$

Ans

$$(2 - 3i)(x + yi) = 4 + i$$

$$2x + 2yi - 3xi - 3yi^2 = 4 + i$$

$$2x - 3y(-1) + (2y - 3x)i = 4 + i$$

$$2x + 3y + (2y - 3x)i = 4 + i$$

$$2x + 3y = 4$$

$$2x = 4 - 3y$$

$$x = \frac{4 - 3y}{2} \quad (1)$$

Put in (2)

$$x = \frac{4 - 3\left(\frac{14}{13}\right)}{2}$$

$$= \frac{52 - 42}{2}$$

$$2y - 3x = 1$$

(2)

$$2y - 3\left(\frac{4 - 3y}{2}\right) = 1$$

$$\frac{4y - 12 + 9y}{2} = 1$$

$$13y - 12 = 2$$

$$13y = 2 + 12$$

$$y = \frac{14}{13}$$

Put in (1)

$$= \frac{\frac{10}{13}}{2}$$

$$= \frac{10}{13 \times 2}$$

$$x = \frac{5}{13}$$

Q.6.(a) Use log tables to find the value of:

$${}^5\sqrt{2.709} \times {}^7\sqrt{1.239}$$

Ans Let $x = {}^5\sqrt{2.709} \times {}^7\sqrt{1.239}$

$$\log x = \log (2.709)^{1/5} \times (1.239)^{1/7}$$

$$\log x = \log (2.709)^{1/5} + \log (1.239)^{1/7}$$

$$= \frac{1}{5} \log (2.709) + \frac{1}{7} \log 1.239$$

$$= 0.0866 + 0.0133$$

$$\log x = 0.0999$$

$$x = \text{Antilog } (0.0999)$$

$$x = 1.2586$$

(b) If $x = 2 + \sqrt{3}$, then find the values of $x - \frac{1}{x}$ and

$$(x - \frac{1}{x})^2.$$

Ans $x = 2 + \sqrt{3}$

(i)

$$x - \frac{1}{x} = ?$$

$$(x - \frac{1}{x})^2 = ?$$

$$\frac{1}{x} = \frac{1}{2 + \sqrt{3}} \times \frac{2 - \sqrt{3}}{2 - \sqrt{3}}$$

$$\frac{1}{x} = \frac{2 - \sqrt{3}}{(2)^2 - (\sqrt{3})^2}$$

$$\frac{1}{x} = \frac{2 - \sqrt{3}}{4 - 3}$$

$$= \frac{2 - \sqrt{3}}{1}$$

$$= 2 - \sqrt{3}$$

(ii)

Subtract eq. (i) and (ii)

$$x - \frac{1}{x} = 2 + \sqrt{3} - 2 + \sqrt{3}$$

$$2 - \frac{1}{x}$$

$$2\sqrt{3} - 2 - \sqrt{3}$$

$$x - \frac{1}{x} = 2\sqrt{3}$$

Taking square on both sides

$$\left(x - \frac{1}{x}\right)^2 = (2\sqrt{3})^2$$

$$\left(x - \frac{1}{x}\right)^2 = 4(3)$$

$$\left(x - \frac{1}{x}\right)^2 = 12$$

Q.7.(a) Factorize: $25x^2 - 10x + 1 - 36z^2$. (3)

Ans $25x^2 - 10x + 1 - 36z^2$
 $= (5x)^2 - 2(5x)(1) + (1)^2 - (6z)^2$
 $= (5x - 1)^2 - (6z)^2$
 $= (5x - 1 + 6z)(5x - 1 - 6z)$

(b) Simplify as rational expression: (4)

$$\left[\frac{x+1}{x-1} - \frac{x-1}{x+1} - \frac{4x}{x^2+1} \right] + \frac{4x}{x^4-1}$$

Ans $\left[\frac{x+1}{x-1} - \frac{x-1}{x+1} - \frac{4x}{x^2+1} \right] + \frac{4x}{x^4-1}$
 $= \left[\frac{(x+1)^2 - (x-1)^2}{(x+1)(x-1)} - \frac{4x}{x^2+1} \right] + \frac{4x}{x^4-1}$
 $= \left[\frac{x^2 + 2x + 1 - x^2 + 2x - 1}{x^2 - 1} - \frac{4x}{x^2+1} \right] + \frac{4x}{x^4-1}$
 $= \left[\frac{4x}{x^2-1} - \frac{4x}{x^2+1} \right] + \frac{4x}{x^4-1}$
 $= \frac{4x(x^2+1) - 4x(x^2-1)}{(x^2-1)(x^2+1)} + \frac{4x}{x^4-1}$

$$\begin{aligned}
 &= \frac{4x^3 + 4x - 4x^3 + 4x}{x^4 - 1} + \frac{4x}{x^4 - 1} \\
 &= \frac{8x}{x^4 - 1} + \frac{4x}{x^4 - 1} \\
 &= \frac{8x + 4x}{x^4 - 1} \\
 &= \frac{12x}{x^4 - 1}
 \end{aligned}$$

Q.8.(a) Find the equation:

$$\frac{x-3}{3} - \frac{x-2}{2} = 1$$

Ans $\frac{x-3}{3} - \frac{x-2}{2} = 1$

$$\frac{2(x-3) - 3(x-2)}{6} = 1$$

$$2x - 6 - 3x + 6 = 6$$

$$-x = 6$$

\Rightarrow

$$x = -6$$

Check:

$$\frac{-6-3}{3} - \frac{(-6-2)}{2} = 1$$

$$\frac{-9}{3} + \left(\frac{8}{2}\right) = 1$$

$$-3 + 4 = 1$$

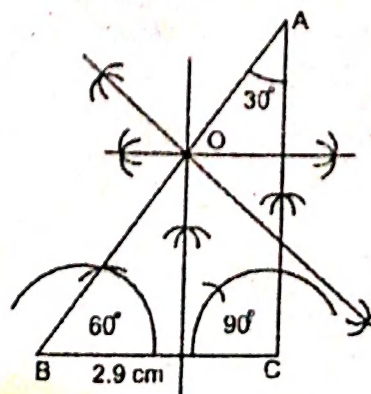
$$1 = 1$$

$$S.S = \{-6\}$$

(b) Construct a $\triangle ABC$ and draw perpendicular bisector of their sides:

$m\angle B = 60^\circ$, $m\angle A = 30^\circ$, $m\overline{BC} = 2.9$ cm

Ans



Q.9. Prove that the bisectors of the angles of a triangle are concurrent. (5)

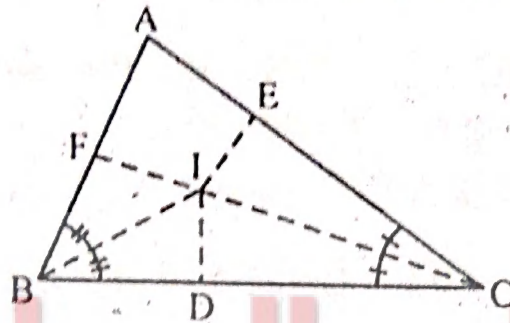
Ans Given:
 $\triangle ABC$

To prove:

The bisectors of $\angle A$, $\angle B$ and $\angle C$ are concurrent.

Construction:

Draw the bisector of $\angle B$ and $\angle C$ which intersect at point I. From I, draw $\overline{IF} \perp \overline{AB}$, $\overline{ID} \perp \overline{BC}$ and $\overline{IE} \perp \overline{CA}$.



Statement

$$\overline{ID} \cong \overline{IF}$$

Reason

(Any point on bisector of an angle is equidistant from its arms)

Similarly,

$$\overline{ID} \cong \overline{IE}$$

$$\therefore \overline{IE} \cong \overline{IF}$$

Each $\cong \overline{ID}$, proved

So, the point I is on the bisector of $\angle A$... (i)

Also the point I is on the bisectors of $\angle ABC$ and $\angle BCA$... (ii)

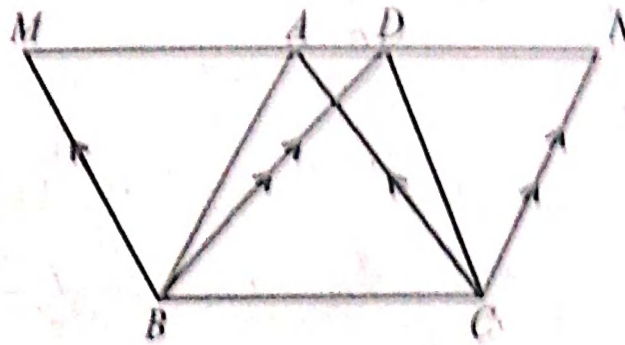
Thus the bisectors of $\angle A$, $\angle B$ and $\angle C$ are concurrent at I.

Construction

{from (i) and (ii)}

Prove that the triangles on equal bases and equal altitudes are equal in area.

Ans



Given: $\triangle ABC$ and $\triangle DBC$ on the same base \overline{BC} , and having equal altitudes.

To prove: Area of $\triangle ABC$ = Area of $\triangle DBC$.

Construction: Draw $\overline{BM} \parallel \overline{CA}$, $\overline{CN} \parallel \overline{BD}$ meeting \overline{AD} produced in M, N.

Proof:

Statements	Reasons
$\triangle ABC$ and $\triangle DBC$ are between the same parallels. Hence $MADN$ is parallel to \overline{BC}	Their altitudes are equal.
Area ($\parallel^{\text{gm}} \text{BCAM}$) = Area ($\parallel^{\text{gm}} \text{BCND}$) (i)	These \parallel^{gm} are on same base \overline{BC} and between the same \parallel^{s} .
But $\triangle ABC = \frac{1}{2} (\parallel^{\text{gm}} \text{BCAM})$ (ii)	Each diagonal of a \parallel^{gm} bisects it into 2 congruent triangles.
And $\triangle DBC = \frac{1}{2} (\parallel^{\text{gm}} \text{BCND})$ (iii)	
Hence, Area ($\triangle ABC$) = Area ($\triangle DBC$)	From (i), (ii) and (iii)